

# Novel Markovian Change Detection Models in Computer Vision

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- 1 Introduction
- 2 Bayesian Foreground and Shadow Detection in Video Scenes
  - Shadow Model
  - Foreground Model
  - Microstructure Model
  - Color Space Selection
  - Evaluation
- 3 Object Motion Detection in Aerial Image Pairs
  - Model Definition
  - Experiments
- 4 Detection of Changes in Built-in Areas
- 5 Publications
- 6 Answers for Reviewer's Comments

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# Introduction

- Image/video based computer systems - digital visual information streams
  - Video surveillance for police
  - Cartography and remote sensing - aerial image analysis
- Change detection - goals
  - Decreasing the number of interesting photos or video frames
  - Extracting object descriptors for higher level image processing modules



Video surveillance



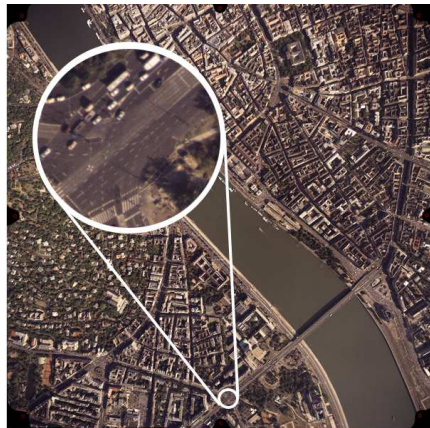
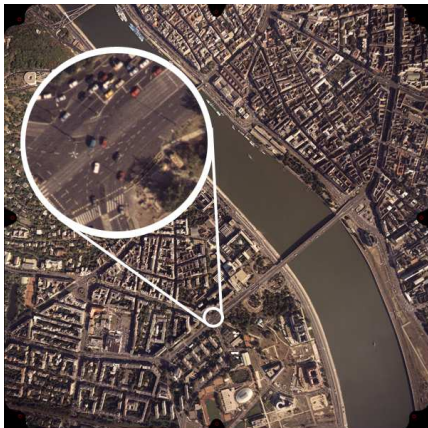
Remote sensing

# Foreground and Shadow Detection in Video Sequences



# Object Motion Detection in Image Pairs Taken by Moving Airborne Vehicles...

- Stereo reconstruction of static scenes



# ... and Processing Low Frame-Rate Aerial Videos

- Large and unpredictable camera motion
- Low frame-rate
- Frame differencing instead of video based techniques



# ... and Processing Low Frame-Rate Aerial Videos





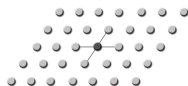
# Detecting Built-in Changes in Image Pairs Taken with Large Time Differences

- Comparing registered photos (Institute of Geoscience, Cartography and Remote Sensing)



# Image Segmentation with Markov Random Fields

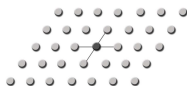
- 2-D pixel lattice  $\rightarrow$  graph:  $S = \{s\}$ 
  - nodes: image points ( $s$  is a pixel)
  - edges: interactions  $\rightarrow$  cliques

Lattice  $S$ 

- Goal: generate a  $K$ -colored segmented image, with a task dependent label set  $L = \{C_1, \dots, C_K\}$ 
  - $\omega_s \in L$ : label of pixel  $s$  which mark its segmentation class
  - Task 1:  $K = 3$ ;  $C_1$ =foreground,  $C_2$ =background and  $C_3$ =shadow.
- Segmentation with Markov Random Fields (MRF):
  - $f_s$ : local feature observed at pixel  $s$  (color, texture etc.)
    - Pixels' feature-values should fit the class models specified by their label
    - Classes are described by feature distributions or probability density functions e.g.  $P(f_s | \omega_s = \text{background})$ .
  - Segmented image is "smooth": we penalize, if two neighboring pixels have different labels

# Image Segmentation with Markov Random Fields

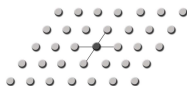
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# Image Segmentation with Markov Random Fields

- Global labeling:  $\underline{\omega} = \{\omega_s | s \in \mathbf{S}\}$
- Observation process:  $\mathcal{F} = \{f_s | s \in \mathbf{S}\}$
- MAP estimation of the optimal global labeling:

$$\hat{\underline{\omega}} = \operatorname{argmax}_{\underline{\omega} \in \Omega} P(\underline{\omega} | \mathcal{F})$$

where  $\Omega$  denotes the set of all the possible global labelings.

- **(Hammersley-Clifford theorem):**  $P(\underline{\omega} | \mathcal{F})$  can be factorized into individual terms whose domains are the cliques of the graph.

$$P(\underline{\omega} | \mathcal{F}) \propto \underbrace{\prod_{s \in \mathbf{S}} P(f_s | \omega_s)}_{P(\mathcal{F} | \underline{\omega})} \cdot \underbrace{\frac{1}{Z} \exp\left(-\sum_{C \in \mathcal{C}} V_C(\underline{\omega})\right)}_{P(\underline{\omega})}$$

- where  $C$  is an arbitrary clique and  $V_C$  is the potential of  $C$ .
- MRF energy function:  $-\log P(\underline{\omega} | \mathcal{F})$

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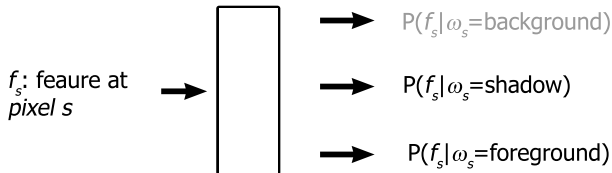
# Thesis Group 1: Bayesian Foreground and Shadow Detection in Video Scenes

- I have worked out a novel spatio-temporal probabilistic model based on MRF for foreground - background separation and cast shadow detection in video frames. I have experimentally shown that the proposed method outperforms the recently published models with the same goals and scene assumptions.



# Bayesian Foreground and Shadow Detection in Video Scenes

- *Likelihood* model of pixel  $s$ :



- Field energy:

$$\sum_{s \in S} -\log P(f_s | \omega_s) + \sum_{r, s \in C} \Theta(\omega_r, \omega_s)$$

$$\Theta(\omega_r, \omega_s) = \begin{cases} -\delta & \text{if } \omega_r = \omega_s \\ +\delta & \text{if } \omega_r \neq \omega_s \end{cases}$$



# Thesis 1.1: Shadow Model

- I have proposed a novel statistical and adaptive color model for detecting cast shadows. I have shown that the procedure is more efficient than using previous approaches if the scene reflection properties are not ideally Lambertian.
  - Photometrical description of the measured color as a function of illumination  $e(\lambda, s)$

$$g(s) = \int e(\lambda, s) \rho(\lambda, s) \nu(\lambda) d\lambda$$

- Former similar models: simplifying assumptions
  - uniform illumination
  - purely Lambertian reflecting surfaces
  - decreased performance in complex scenarios
- Proposed approach: global statistical characterization of pixel level, physical shadow descriptors

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# Foundations of the Shadow Model

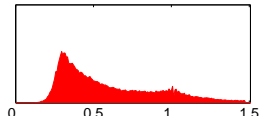
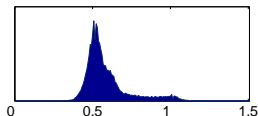
- Constant ratio model (noise sensitive):

$$g_{\text{shadow}}(s) = A \cdot g_{\text{background}}(s)$$

- Proposed approach (1D visualization):

$$\psi(s) = g(s)/g_{\text{background}}(s)$$

- spatiotemporal histograms of **shadow-** and **foreground**  $\psi(s)$  values:

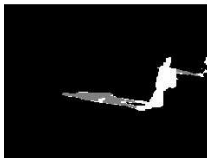


- Approximating the shadow domain: Gaussian density functions
- Color images:  $\bar{\psi}(s)$  3D vector, 3D Gaussian density

# Experimental Validation of the Shadow Model



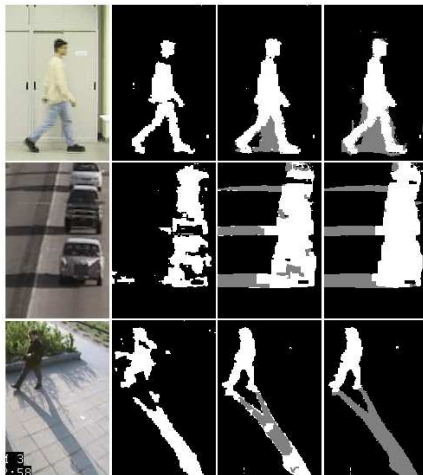
Constant ratio:



Proposed:



Illumination invariant    Constant ratio    Proposed



## Thesis 1.2: Foreground Model

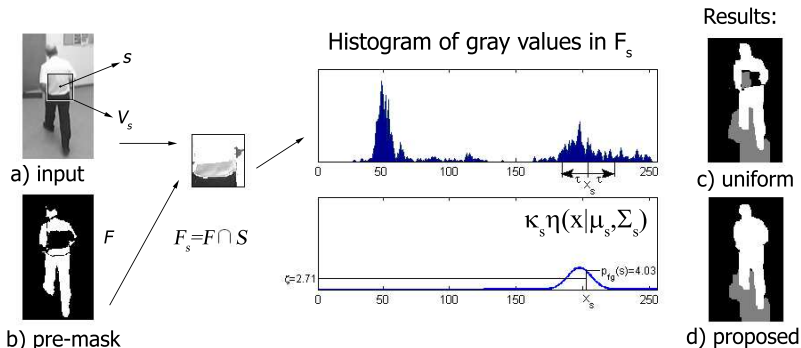
- A novel foreground description has been given based on spatial statistics of the nearby pixel values. I have shown that the introduced approach enhances the detection of background or shadow-colored object parts, even in low and/or unsteady frame rate videos.
  - Predicting the colors in the foreground
    - irrelevant temporal statistics
    - uniform color model - weak to detect fine differences
    - spatial statistics: in the neighborhood of a foreground pixel other foreground pixels are expected with similar color

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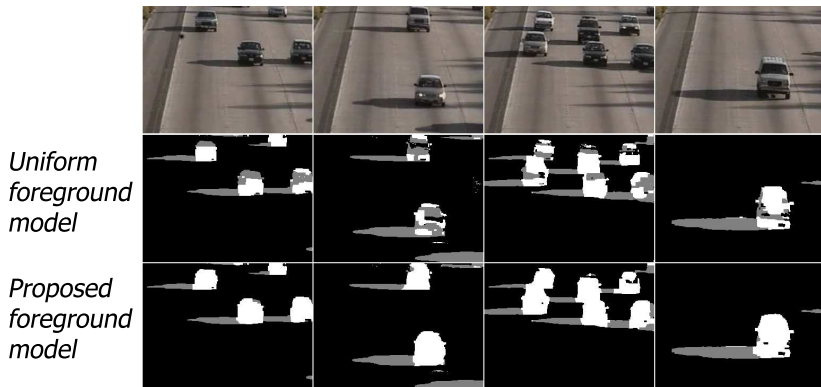
# Introduction of the Foreground Model

- Estimation of the color statistics of the probably foreground pixels in each neighborhood



# Introduction of the Foreground Model

- Example results:





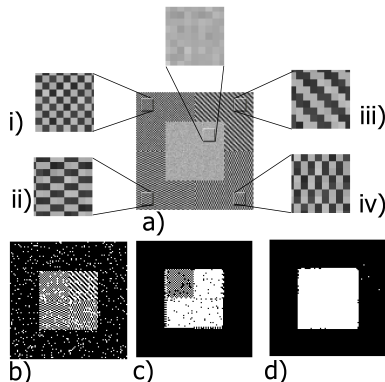
## Thesis 1.3: Microstructure Model

- I have given a probabilistic model of the microstructural responses in the background and in the shadow. Thereafter, I have completed the MRF segmentation model with microstructure analysis. The proposed adaptive kernel selection strategy considers the local background properties. I have shown via synthetic and real-world examples, that the improved framework outperforms the purely color based model, and methods using a single kernel.
  - goal: considering textural differences in the separation
  - texture distribution parameters can be analytically estimated

# Effects of the Microstructure Model

## Synthesised Example

- Input image - Fig. a)
  - homogenous but noisy foreground (light rectangle in the middle)
  - inhomogeneously textured background
- Results:
  - b) only intensity based separation
  - c) intensity + edge based model
  - d) proposed adaptive kernel selection

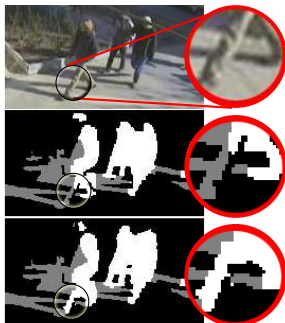


# Effects of the Microstructure Model

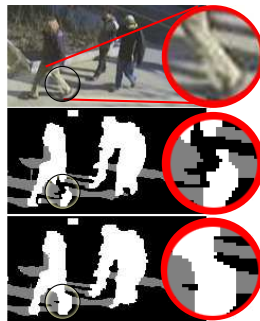
## Real Image Examples

- Improvements in regions of finely textured details

Without texture analysis



With texture analysis



# Thesis 1.4: Color Space Selection for Shadow Detection

- I have experimentally shown that among the widespread color spaces, the CIE  $L^*u^*v^*$  model is the best for cast shadow detection, both using an elliptical separation in the space of the pixel-level descriptors and regarding a color space independent extension of the proposed MRF-segmentation model.

# Shadow Descriptors in Different Color Spaces

Color space:

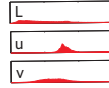
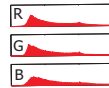
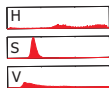
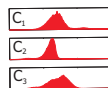
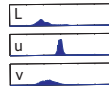
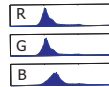
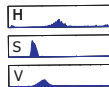
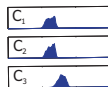
 $C_1C_2C_3$ 

HSV

RGB

CIE  $L^*u^*v^*$ 

1-D histograms:



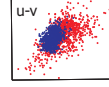
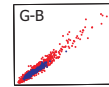
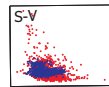
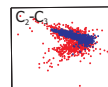
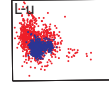
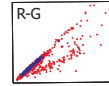
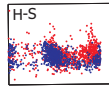
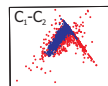
Shadow:



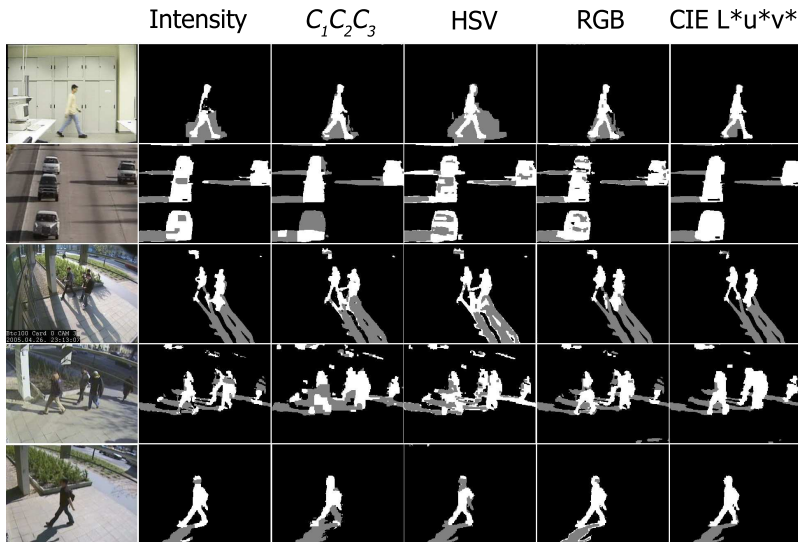
Foreground:



2-D scatter plots:

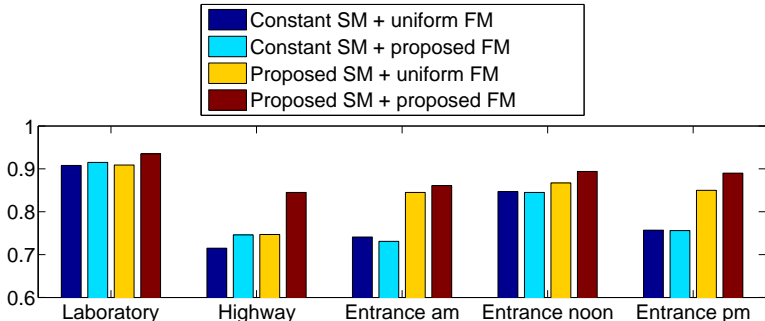


# Comparing Color Spaces in the Proposed MRF Framework



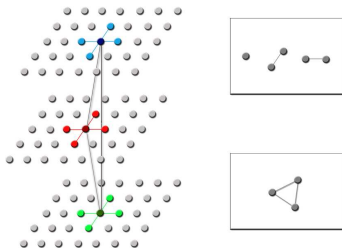
# Quantitative Evaluation of the Results from Thesis Group 1.

- 2 benchmark sequences and 3 real surveillance videos, in aggregate 861 evaluated frames
- Metric:  $F$ -measure (harmonic mean of recall and precision of foreground detection)
- Notation: SM = shadow model, FM = foreground model



## Thesis Group 2: Three-Layer Markovian Models

- I have developed novel three-layer MRF models for object motion detection in unregistered aerial image pairs and built-in change detection in aerial photos captured with several years time difference. I have experimentally validated the proposed models.





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## Thesis 2.1

- I have developed a novel statistical model for object motion detection in image pairs captured by moving airborne vehicles. I have experimentally shown that the proposed approach outperforms previous models which use purely linear image registration techniques or local parallax removal.

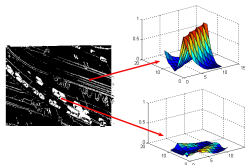


# Feature Extraction

- 1. feature: gray level difference

$$d(s) = \tilde{x}_2(s) - x_1(s)$$

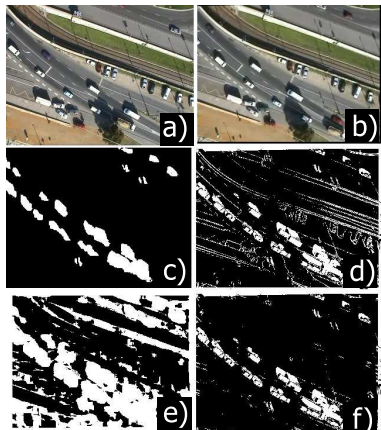
- 2. feature: local correlation peak value  $c(s)$



- Pixel  $s$  belongs to background, if:

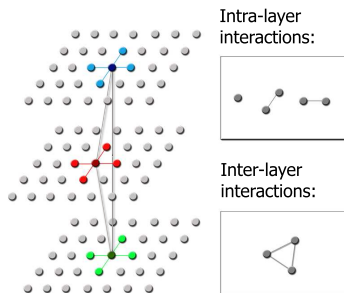
$$|d(s)| < T_1 \text{ OR } c(s) > T_2$$

- Spatial smoothing is necessary - composite Markovian model due to feature integration



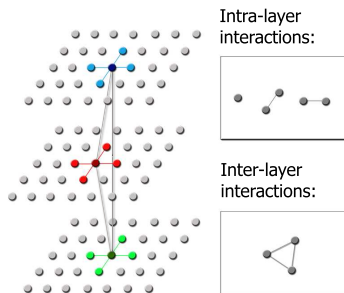
# 3-Layer Markov Random Field Model

- Classes: object motion (i.e. foreground), background
- Layers:
  - 1. **observation layer**: MRF based on intensity difference
  - **Segmentation layer**: final result by feature integration
  - 2. **observation layer**: MRF based on the correlation peak feature
- Singletons
  - Labels in the observation layers should be consistent with local features  $d(s)$  resp.  $c(s)$
- Intra layer connections
  - Smooth segmentation in each layer
- Inter layer interactions
  - Synchronizing the segmentations by label fusion.



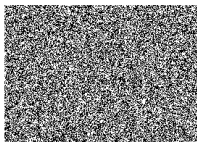
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# Field Energy Optimization

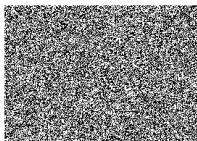
$$\hat{\omega} = \operatorname{argmin}_{\omega \in \Omega} \left\{ - \sum_{s \in S} \log P(d(s) | \omega(s^d)) - \sum_{s \in S} \log P(c(s) | \omega(s^c)) + \right. \\ \left. + \sum_{\{r^i, s^i\} \in \mathcal{C}_2} \beta \cdot \delta(\omega(r^i), \omega(s^i)) + \sum_{s \in S} I(\omega(s^d), \omega(s^c), \omega(s^*)) \right\}$$



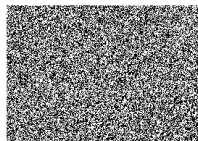
d(s) layer



ground truth



c(s) layer



result layer

# Test Datasets and Reference Methods

- Database: 83 image pairs from 3 test sets
- Comparison to manual segmentation
- Metric:  $F$ -measure
- Reference methods:
  - FFT similarity matching
  - Method of Farin and With , ICIP 2005<sup>1</sup>
  - Supervised affine matching

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<sup>1</sup>D. Farin and P. With, "Misregistration Errors in Change Detection Algorithms and How to Avoid Them," in *Proc. International Conference on Image Processing (ICIP)*, vol. 2, pp. 438-441, Genoa, Italy, Sept. 2005.

# Results

First image



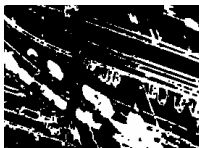
Second image



Ground truth



FFT similarity



Supervised affine



Farin's method



3-layer MRF





# Results

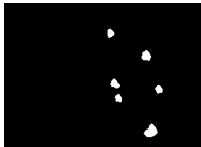
First image



Second image



Ground truth



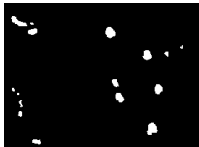
FFT similarity



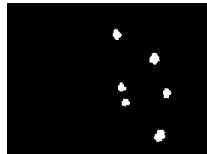
Supervised affine



Farin's method



3-layer MRF

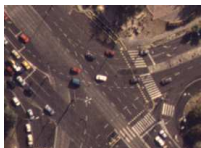


# Results

First image



Second image



Ground truth



FFT similarity



Supervised affine



Farin's method



3-layer MRF



# Results in 'Balloon 1' Test Set



Video frame



FFT similarity



Farin's method



Ground truth

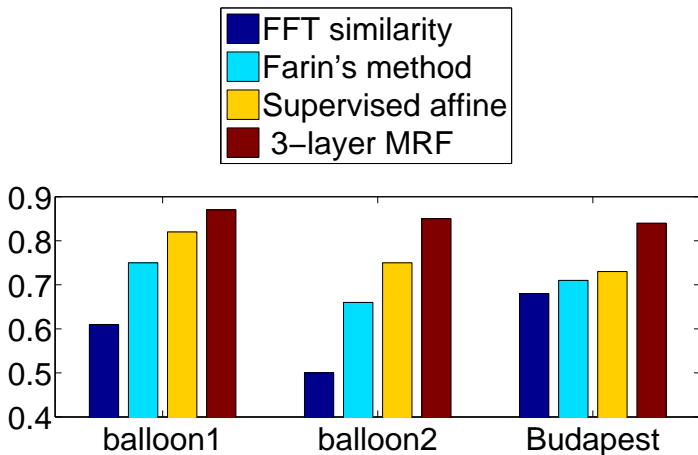


Supervised affine



3-layer MRF

# Quantitative Results ( $F$ -measure)



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## Thesis 2.2

- I have developed a Markovian framework for structural change detection in aerial photos captured with significant time difference. I have shown through an application on built-in change detection that connecting the segmentations of the different images via pixel-level links results in an efficient region based change detection method, which is robust against the noise and incompleteness of the class descriptors.



# Structural Change Detection in Aerial Images Captured with Large Time Differences

- Preliminary registered aerial photos
- 5-20 years difference
- Pixel-level comparison is irrelevant
- Region based change detection
  - Segmenting the images with the same clusters: built-in and natural areas
  - Detecting regions with changed clusters



# Feature Selection for Built-in Change Detection

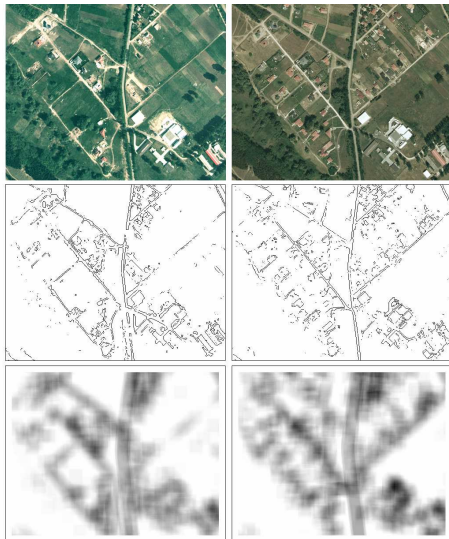
- “Edge-density” textural descriptor
- Edge map:

$$E = \{E(s) | s \in S\}$$

- Edge density map:

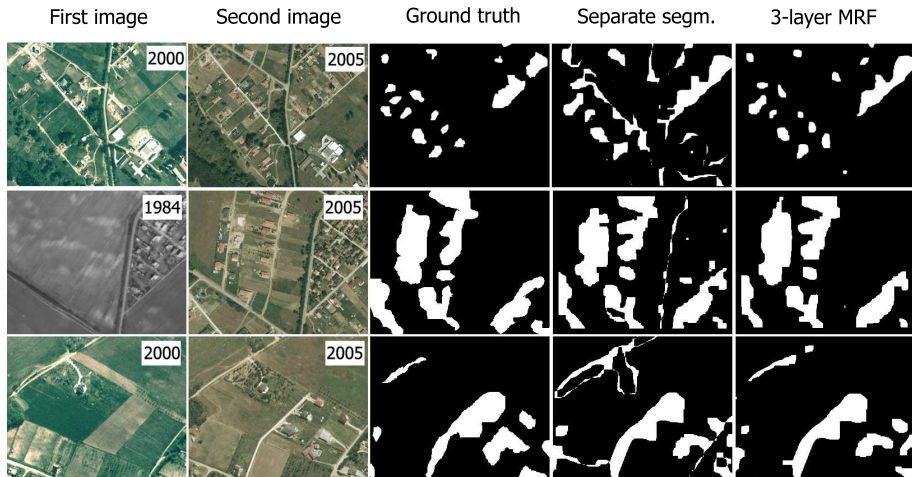
$$T(s) = \frac{1}{(2W + 1)^2} \sum_{|r-s| \leq W} E(r)$$

- In built-in areas the edge density is high
- Region borders are ambiguous





# Built-in Change Detection: Results



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# Journal Publications

- Cs. Benedek and T. Szirányi: “Bayesian Foreground and Shadow Detection in Uncertain Frame Rate Surveillance Videos”, *IEEE Transactions on Image Processing*, vol. 17, no. 4, pp. 608 - 621, April 2008. (IF: 2.715)
- Cs. Benedek and T. Szirányi: “Study on Color Space Selection for Detecting Cast Shadows in Video Surveillance,” *International Journal of Imaging Systems and Technology, Special Issue on Applied Color Image Processing*, vol. 17, no. 3, pp. 190-201, Wiley, 2007 (IF: 0.983)

## International Conference Publications 2/1

- Cs. Benedek , T. Szirányi, Z. Kato and J. Zerubia: “A Multi-Layer MRF Model for Object-Motion Detection in Unregistered Airborne Image-Pairs,” in Proc. *IEEE International Conference on Image Processing (ICIP)*, 2007
- Cs. Benedek and T. Szirányi: “Markovian Framework for Foreground-Background-Shadow Segmentation of Real World Video Scenes”, *Asian Conference on Computer Vision (ACCV), Lecture Notes in Computer Science, Springer*, 2006
- Cs. Benedek and T. Szirányi: “Color Models of Shadow Detection in Video Scenes”, in Proc. *International Conference on Computer Vision Theory and Applications (VISAPP)*, 2007
- Cs. Benedek and T. Szirányi: “Markovian Framework for Structural Change Detection with Application on Detecting Built-in Changes in Airborne Images,” in Proc. *IASTED International Conference on Signal Processing, Pattern Recognition and Applications (SPPRA)*, 2007

## International Conference Publications 2/2

- D. Szolgay, Cs. Benedek and T. Szirányi: “Fast Template Matching for Measuring Visit Frequencies of Dynamic Web Advertisements”, *International Conference on Computer Vision Theory and Applications (VISAPP)*, 2008
- Z. Szlávik, L. Havasi, Cs. Benedek and T. Szirányi: “Motion-based Flexible Camera Registration”, *IEEE International Conference on Advanced Video and Signal-Based Surveillance (AVSS)*, 2005
- Z. Szlávik, T. Szirányi, L. Havasi and Cs. Benedek: “Optimizing of Searching Co-Motion Point-Pairs for Statistical Camera Calibration”, *IEEE International Conference on Image Processing (ICIP)*, 2005
- Z. Szlávik, T. Szirányi, L. Havasi and Cs. Benedek: “Random Motion for Camera Calibration”, *European Signal Processing Conference (EUSIPCO)*, 2005
- L. Havasi, Z. Szlávik, Cs. Benedek and T. Szirányi, “Learning human motion patterns from symmetries”, *ICML Workshop on Machine Learning for Multimedia*, 2005
- L. Havasi, Cs. Benedek, Z. Szlávik and T. Szirányi: “Extracting Structural Fragments from Images Showing Overlapping Pedestrians”, *IASTED International Conference on Visualization, Imaging, and Image Processing*, 2004

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  - Zoltán Szlávik, László Havasi, István Petrás, Levente Kovács
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- Collaborators
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- Reviewers of the Dissertation
- All colleagues at SZTAKI, PPKE and INRIA institutes
- My Lívi, family & friends

Thank you for your attention

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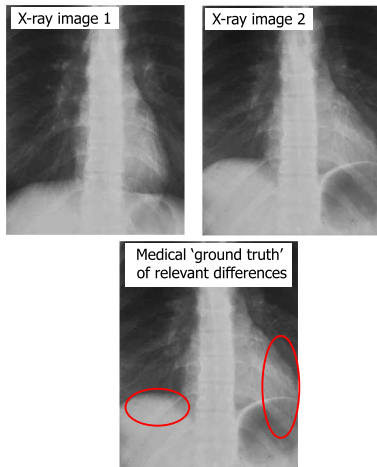


## Kernel Size Used for Texture Analysis

- Question: Can be the quality of segmentation enhanced by using  $5 \times 5$  or larger kernels to compute the microstructural responses instead of  $3 \times 3$  kernels?
- Effects of using larger kernels:
  - ⊕ improved detection of the internal parts of object/background regions
  - ⊖ increased artifacts appear near to the class-boundaries
- Optimal kernel size depends on:
  - Image resolution
  - Size of objects
- In  $320 \times 240$  video frames  $3 \times 3$  proved to be a good compromise.

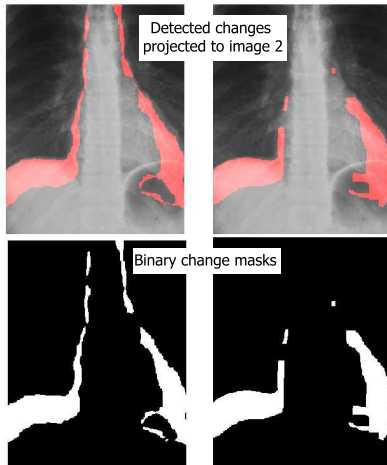
# Segmenting X-ray Images with the Proposed Markovian Structure

Input images and 'ground truth'



Change detection results with

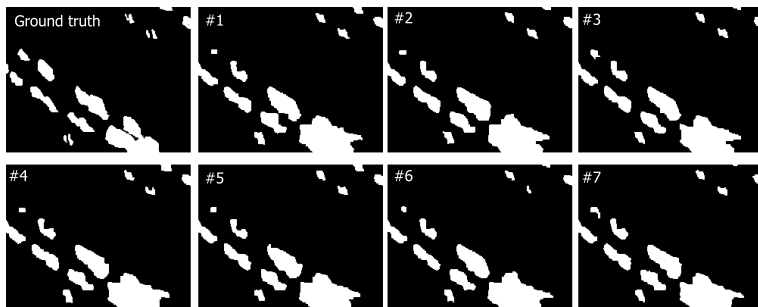
a) separate segm.      b) three-layer MRF



# Experimental Validation of the Stochastic Optimizer

Repeatability of the experiments for the 3-layer MRF model

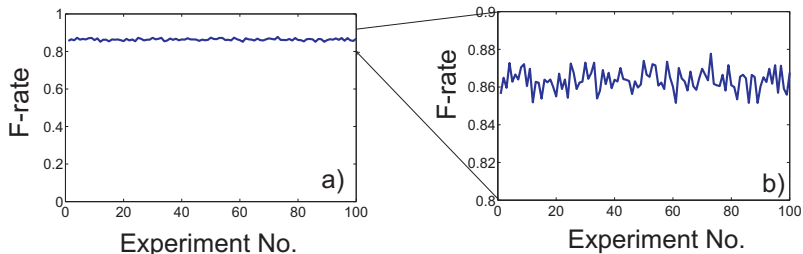
- Qualitative results of 7 different pseudo-stochastic optimization experiments with the *same* image pair, *same* parameters and relaxation settings, but *different* seeds for the RANDOMIZE calls



# Experimental Validation of the Stochastic Optimizer

## Repeatability of the experiments for the 3-layer MRF model

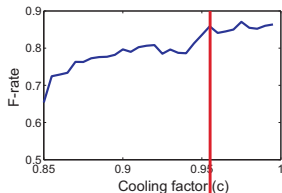
- Quantitative results of 100 different pseudo-stochastic optimization experiments with the *same* image pair, *same* parameters and relaxation settings, but *different* seeds for the RANDOMIZE calls
  - measured mean value of  $F$ -rates: 0.8635
  - measured standard deviation: 0.0057



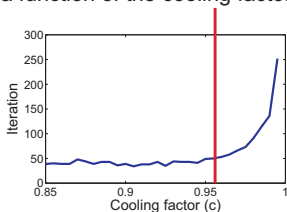
# Experimental Validation of the Stochastic Optimizer

Effects of changing the cooling factor and the iteration number

Performance as a function of the cooling factor ( $c$ ), obtained at convergence:



Number of MMD iterations till convergence as a function of the cooling factor ( $c$ ):



Study on the convergence speed obtained at  $c=0.96$ :

